AZADAN BHAGWAGAR 14000

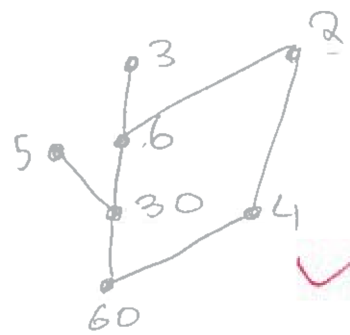
Exam III: MTH 213, Spring 2018

Ayman Badawi

$Score = \frac{37}{37}$

QUESTION 2. $A = \{2, 3, 4, 5, 6, 30, 60\}$. Define $\leq$ on A such that $\forall a, b \in A$, $a \leq b$ if and only if $a = bc$ for some $c \in N^*$. Then $(A, \leq)$ is a partially ordered set (DO NO SHOW THAT)

(i) **(4 points)** Draw the Hassee diagram of such relation



(ii) **(3 points)** By staring at the Hassee diagram, If possible, find

a. $5 \wedge 6$ 30

b. $6 \wedge 4$ 60

c. $6 \vee 3$ 3

d. $30 \vee 60$ 30

e. Is there a $c \in A$ such that $a \leq c$ for every $a \in A$? If yes, find c Does not exist

f. Is there an $m \in A$ such that $m \leq a$ for every $a \in A$? If yes, find m 60

QUESTION 3. (10 points)

(i) Let F be a set with 7 elements, and let $H = \{d \subset F \mid |d| = 4\}$. Find $|H|$ (i.e., find the cardinality of H)

7C4 ✓

(ii) How many 5-digit even integers greater that 60000 can be formed using the digits (1, 2, 3, 4, 5, 6, 7) such that the second and the third digit must be odd integer.

2C1 ✓ 1C1 × 4C1 × 4C1 × 7C1 × 3C1 = 672

(1st) (2nd) (3rd) (4th) (5th)

(iii) There are 7 dots randomly placed on a circle such that exactly 4 of them are red and the remaining three dots are green. How many triangles can be formed within the circle (i.e., inside the circle) such that each triangle has exactly two green vertices?

3C2 × 4C1 ✓

(iv) 491 kids are in a gathering, all of them were born between 2010-2014. It is observed that more than 60% of them are girls. Then there exist at least n kids who were born in the same month and in the same year. What is the maximum value of n?

2014 − 2010 + 1 = 5

Dom Codomain

491 (month, year) 5×12

By Pigeon-hole Principle:

$n = \left\lceil \frac{491}{60} \right\rceil = 9$ ✓

(v) In the above question, there is a month and a year between 2010-2014 such that at most m kids were born in that month and in that year. Find the minimum value of m.

$m = \left\lfloor \frac{491}{60} \right\rfloor = 8$ ✓

QUESTION 4. (4 points)

Given $S = \{1, 2, 3, 4, 5, 6, 7, 8\}$. Let f be a bijective function from S onto S such that

$$f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 7 & 1 & 5 & 8 & 6 & 4 & 2 & 3 \end{pmatrix}$$

(i) Find f^2 (i.e., find fof). (Note that by staring at f, we understand that $f(1) = 7, ..., f(8) = 3$)

$$f^2 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 2 & 7 & 6 & 3 & 4 & 8 & 1 & 5 \end{pmatrix}$$ ✓

(ii) Find the least positive integer n such that $f^n = I$, where I is the identity map (i.e., $I(a) = a$ for every $a \in S$)

$f = (1,7,2)(3,5,6,4,8)$; $n = LCM[3,5] = 15$ ✓

QUESTION 5. (2 points) Let $M = Q \cap (-1, 0)$. Is M countable? Is $|M| = |Q|$? explain briefly. Q is countable.

There are infinitely many rational numbers between −1 and 0. ∴ $Q \cap (-1,0)$ is a subset of Q with cardinality infinite countable. So Yes, M is countable, $|M| = |Q|$.

QUESTION 6. (3 points) Given $f : [-2, \infty) \to [-4, \infty)$ is a function such that $f(x) = x^3 + \sqrt{x + 11} + e^{(x+2)}$. Use mathematical argument and convince me that $\exists! m \in [-2, \infty)$ such that $f(m) = 0$.

$f(-2) = (-2)^3 + \sqrt{9} + e^0 = -4$; $\lim_{x \to \infty} f(x) = \infty + \infty + \infty = \infty$;

range = codomain

f is continuous, ∴ f is onto.

$f'(x) = 3x^2 + \frac{1}{2(x+11)} + e^{(x+2)}$, $\underset{-2 \qquad\qquad \infty}{\overset{+}{\vdash\!\!-\!\!-\!\!-\!\!\dashv}}$, $f'(x)$ is always positive over domain ∴ $f(x)$ is increasing only ∴ $f(x)$ is 1-1. ok but why? –

Since $f(x)$ is onto, there exist 'm' in domain s.t. $f(m) = 0$ because $0 \in [-4, \infty)$, and since $f(x)$ is also 1-1, this m is unique.

$\phi($

$=$

$\phi(8$

resu

$=$

Faculty information

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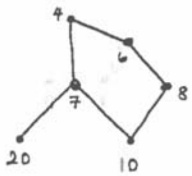
$((x+11)^{1/2}$

$(1/2)(x+11)^{-1/2}$

QUESTION 4. $A = \{4, 6, 7, 8, 10, 20\}$. Define $\leq$ on A such that $\forall a, b \in A$, $a \leq b$ if and only if $b - a \in \{0, -2, -3, -4, -6, -13, -16\}$. Then $(A, \leq)$ is a partially ordered set (DO NOT SHOW THAT)

(i) **(4 points)** Draw the Hassee diagram of such relation

- 4 "≤" : (4).
- 6 "≤" : 4, (6).
- 7 "≤" : 4, (7).
- 8 "≤" : 4, 6, (8).
- 10 "≤" : 4, 6, ~~7~~, 8, (10).
- 20 "≤" : 4, 7, (20).



(ii) **(3 points)** By staring at the Hassee diagram, if possible, find

a. $20 \vee 10$ = 7.

b. $7 \wedge 6$ = 10.

c. $20 \vee 8$ = 4

d. $20 \vee 4$ = 4.

e. Is there a $c \in A$ such that $a \leq c$ for every $a \in A$? If yes, find c ⇒ Yes, c = 4.

f. Is there an $m \in A$ such that $m \leq a$ for every $a \in A$? If yes, find m ⇒ NO, DNE.

QUESTION 5. (3 points) Convince me that $|(-\infty, 0]| = |(-5, 4]|$ by constructing a bijective function between the two sets.

Let $f : (-\infty, 0] \longrightarrow (-5, 4]$

$\therefore f(x) = 9e^x - 5$.

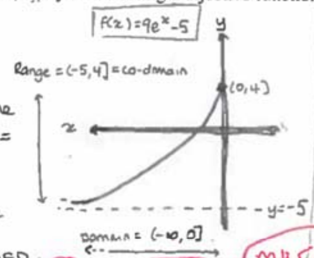
From graph, it is clear that f(x) is one-to-one (since function is increasing) and onto (since range = co-domain).

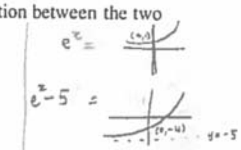
Hence f is a bijective function.

We know from fact that, if a bijective function can be built, the domain & codomain have same cardinality.

$\therefore |(-\infty, 0]| = |(-5, 4]|$ QED.

$\boxed{f(x) = 9e^x - 5}$





$me^0 - 5 = 4$

$m - 5 = 4 \Rightarrow m = 9$.

QUESTION 6. (6 points)

(i) Let F be a set with 4 elements, Consider the power set of A, $P(A)$, and let $H = \{d \subset F \mid |d| = 3\}$. Find $|H|$ (i.e., find the cardinality of H)

irrelevant

my fault

|F| = 4.

H is a set containing all subsets of F for which cardinality of subset = 3.

$\therefore$ |H| = # of all possible subsets d, where |d| = 3. (order in subsets not important)

Hence |H| = 4C3 = 4.

6

my fault

supposed to be P(A), so 16C3

(ii) How many 4-digit even integers greater than 5300 can be formed using the digits (2, 3, 4, 5, 6, 7, 8) such that the third digit must be an even integer.

1^{st} = 4C1 (>5000)

2^{nd} = 6C1 (>300)

3^{rd} = 4C1 (even; {2, 4, 6, 8})

4^{th} = 4C1 (even integer).

$\therefore$ Total possibilities = 4C1 × 6C1 × 4C1 × 4C1 = 384.

(-1) 1×6×4×4 + 3×7×4×4

(iii) There are 432 balls and there are 9 holes (very deep holes). The holes are labeled A, A, A, B, B, B, C, C, C. 123 balls must be placed in A-holes (i.e., maybe all of them in one A-hole, or in two A-holes or in three A-holes), 200 balls must be placed in B-holes (see my earlier comment), and the remaining balls must be placed in C-Holes (again, see my earlier comment). Then there are at least n balls that are placed in the same hole (such hole could be an A-hole, or a B-hole, or a C-hole). What is the maximum value of n?

|Domain A| = 123. |Co-domain A| = 3.

|Domain B| = 200. |Co-domain B| = 3.

|Domain C| = 432 − 323 = 109. |Co-domain C| = 3.

⇒ least balls in A = $\lceil \frac{123}{3} \rceil$ = 41.

least balls in B = $\lceil \frac{200}{3} \rceil$ = 67

least balls in C = $\lceil \frac{109}{3} \rceil$ = 37.

$\therefore$ least balls placed in same hole of any kind = min[41, 67, 37]

$\therefore \boxed{n = 37}$.

Answer 67

QUESTION 7. (2 points) Find 8^{24002} (*mod* 35)

Fact: If gcd(a, n) = 1 then $a^{\phi(n)}$ (mod n) = 1

Summer

QUESTION 8. (3 points)
Given $S = \{1,2,3,4,5,6,7,8\}$. Let f be a bijective function from S onto S such that

$$f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 7 & 6 & 5 & 3 & 4 & 2 & 1 & 8 \end{pmatrix}$$

(i) Find f^2 (i.e., find $f o f$). (Note that by staring at f, we understand that $f(1) = 7, \ldots, f(8) = 8$)

$F^2 = FoF = F(F(2))$.

$$\therefore F^2 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 1 & 2 & 4 & 5 & 3 & 6 & 7 & 8 \end{pmatrix}$$

(ii) Find the least positive integer n such that $f^n = I$, where I is the identity map (i.e., $I(a) = a$ for every $a \in S$)

f as disjoint cycle $= \underbrace{(1\ 7)}_{2\text{-cycle}}\underbrace{(2\ 6)}_{2\text{-cycle}}\underbrace{(3\ 5\ 4)}_{3\text{-cycle}}$.

$\therefore n = \text{Lcm}[2,2,3] = \underline{\underline{6}}$. $\therefore \underline{\underline{F^6 = I}}$.

QUESTION 9. (3 points) Let $A = \{0,1,2,3,4\}$ and consider the following equivalence classes of an equivalence relation "=" on A: $[0] = \{0,3,4\}$, $[1] = \{1\}$, $[2] = \{2\}$. Note that the definition of "=" is not given here. Write down the elements (explicitly) of "=" as a subset of $A \times A$.

$[0] = \{0,3,4\}$.
$[1] = \{1\}$
$[2] = \{2\}$.

$\therefore$ Elements of "=" as subset of $A \times A$
$= (0,0), (0,3), (0,4), (3,0), (3,3), (3,4), (4,0), (4,3), (4,4), (1,1), (2,2)$.

QUESTION 10. (4 points) Let $A = \{1,2,4,7,8,11,13,14\}$ and let $H = \{1,4,7,13\}$. Define "=" on A such that $\forall a,b \in A$, $a" = "b$ if and only if $ab\ (mod 15) \in H$. Then "=" is an equivalence relation. Do not show that.

(i) Does $3 = 7$? why? $\Rightarrow$ If 3"="7, this means $(3)(7)(\text{mod } 15) \in H$. $(3)(7)(\text{mod } 15) = 21(\text{mod } 15) = 6 \notin H$.
Hence, $\boxed{3 " \neq " 7}$.

(ii) Does $11 = 14$? why? If 11"="14, then $(11)(14)(\text{mod } 15) \in H$. $(11)(14)(\text{mod } 15) = 154(\text{mod } 15) = 4 \in H$.
Hence $\boxed{11 "=" 14}$.

(iii) Find all equivalence classes of A

$[1] = \{1,4,7,13\}$.
$[2] = \{2,8,11,14\}$.

$= 34a + 126b$

$2 = 10 - 8$
$= 10 - (4 \times 2)$
$= 10 - ((24-20) \times 2)$
$= 10 - (24 \times 2) + (20 \times 2)$
$= (10 \times 1) - (24 \times 2) + (10 \times 4)$
$= (10 \times 5) - (24 \times 2)$.

$\Rightarrow$ ② $\overline{4}$ ... 2, -4, 0 ← stop

$= ((34-24) \times 5) - (24 \times 2)$
$= (34 \times 5) - (24 \times 5) - (24 \times 2)$
$= (34 \times 5) - (24 \times 7)$
$= (34 \times 5) - ((126-102) \times 7)$
$= (34 \times 5) - (126 \times 7) + (102 \times 7)$
$= (34 \times 5) - (126 \times 7) + (34 \times 3) \times 7$
$= (34 \times 5) - (126 \times 7) + (34 \times 21)$
$= (34 \times 26) + (126 \times -7)$

$\therefore$ $\boxed{\begin{array}{l} m = 2. \\ a = 26. \\ b = -7 \end{array}}$.

Exam II: MTH 213, Spring 2018

Mariam Reda

Ayman Badawi

Score = $\frac{33}{33}$

QUESTION 1. (6 points)
Imagine: The following Algorithm segment. Find the exact number of additions, multiplications, and subtractions that will be performed when the algorithm is executed. Then find the complexity of the Algorithm segment.

$m = 7; s = 3$
For $k := 1$ to $\lfloor \frac{n}{2} \rfloor$
$L = k * m + s^2 + 2 * s - 2$
For $i := 1$ to $(k + 2)$
$W = s^4 + 3 * k + m^3 + i - 8$
next i
next k

*Assume n is odd.

Outer loop:
- Iterates from $k=1$ to $\lfloor \frac{n}{2} \rfloor$
  If n is odd: $\lfloor \frac{n}{2} \rfloor = \frac{n-1}{2}$
  ~~If n is even: $\lfloor \frac{n}{2} \rfloor = \frac{n}{2}$~~
- Total number of times code is executed:
  ~~Even: $\frac{n}{2} - 1 + 1 = \frac{n}{2}$~~
  Odd: $\frac{n-1}{2} - 1 + 1 = \frac{n-1}{2}$
- Number of operations per single iteration of outer loop = 6
  $[L = k*m + s*s + 2*s - 2]$
- Total number of operations in outer loop:
  ~~$= 6(\frac{n}{2}) = 3n$ [even]~~
  $= 6(\frac{n-1}{2}) = \boxed{3n-3}$.

Inner loop:
- Iterates from $i = 1$ to $(k+2)$.
- Number of operations per single iteration of inner loop = 10
  $[s*s*s*s + 3*k + m*m*m + i - 8]$

- Number of ~~iterations~~ operations of inner loop for each outer loop:
  $k=1$, $k=2$, $k=3$, $k=\frac{n-1}{2}$
  $\underbrace{10\times3,\ 10\times4,\ 10\times5, \ldots, 10\times(\frac{n-1}{2}+2)}_{\text{arithmetic}}$

$$= \frac{(\frac{n-1}{2})[(10\times3) + (10(\frac{n-1}{2}+2))]}{2}$$
$$= \frac{(\frac{n-1}{2})(5n+45)}{2}$$
$$= \frac{5n^2+40n-45}{4}.$$

not needed

∴ Total operations for algorithm
$$= \boxed{\frac{5n^2+40n-45}{4} + 3n - 3}$$
$[= S_n]$
∴ complexity $= O(S_n) = n^2$.

QUESTION 2. (2 points)

(i) $O(\frac{x^{0.9}+3x^{1.2}-5}{x+7})$ equals to $\frac{O(x^{0.9}+3x^{1.2}-5)}{O(x+7)} = \frac{x^{1.2}}{x^1} = x^{1.2-1} = x^{0.2}$

(ii) $O(\sqrt[5]{x} + x^2 + 1)(x^2 - x^{7/2} - 2)$ equals to
$O(\sqrt[5]{x}+x^2+1) \cdot O(x^2 - x^{7/2} - 2) = n^2 \cdot n^{7/2} = x^{2+7/2} = x^{11/2}$.

(?) **QUESTION 3. (6 points)** Imagine we have a sequence $\{a_n\}_{n=0}^{\infty}$, where $a_0 = 4$, $a_1 = 2$, and $a_n = 4a_{n-1} - 4a_{n-2} + 3n + 1$. Find a general formula for a_n. Then find a_6.

$a_n = 4a_{n-1} - 4a_{n-2} + 3n + 1$.
Let $b_n = 4b_{n-1} - 4b_{n-2}$
⇒ There exists some α s.t.
$\alpha^n = 4\alpha^{n-1} - 4\alpha^{n-2}$
$\Downarrow (\div \alpha^{n-2})$
$\alpha^2 = 4\alpha - 4$
$\alpha^2 - 4\alpha + 4 = 0$
$(\alpha-2)(\alpha-2) = 0$.
∴ $\alpha = 2$ (repeated root).

⇒ a_n can be written as
$a_n = 4c_1(2)^n - 4c_2 n(2)^n + 3n + 1$.
- Let $n = 0$:
  $a_0 = 4$
  $= 4c_1(2)^0 - 4c_2(0)(2)^0 + 3(0) + 1$
  ∴ $4 = 4c_1 + 1$. ①
- Let $n = 1$:
  $a_1 = 2$
  $= 4c_1(2)^1 - 4c_2(1)(2)^1 + 3(1) + 1$
  ∴ $2 = 8c_1 - 8c_2 + 4$. ②

From ①: $\frac{4-1}{4} = c_1$ ∴ $c_1 = \frac{3}{4}$
From ②: $2 = 8(\frac{3}{4}) - 8c_2 + 4$
$2 - 10 = -8c_2$
$\frac{-8}{-8} = c_2$ ∴ $c_2 = 1$.

∴ General formula
$a_n = 4(\frac{3}{4})(2)^n - 4(1)n(2)^n + 3n + 1$
∴ $a_n = 3(2)^n - 4n(2)^n + 3n + 1$.

⇒ a_6. Let $n = 6$.
∴ $a_6 = 3(2)^6 - 4(6)(2^6) + 3(6) + 1 = -1325$.

QUESTION 6. (i) Let $A = \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$ and $B = \{0, 3, 6\}$. Define "=" on A so that $a = b$ if $(a - b)\ mod(9) \in B$

[redacted]

b. **(3 points)** Find all equivalence classes of A.

$\bar{0} = \{0, 3, 6\}$

$\bar{1} = \{1, 4, 7\}$

$\bar{2} = \{2, 5, 8\}$

assume $a = 6$ $b = 3$ $c = 0$

$a - b = 6 - 3 = 3$ mod $9 = 3 \in B$

$b - c = 3 - 0 = 3$ mod $9 = 3 \in B$

$a - c = 6 - 0 = 6$ mod $9 = 6 \in B$

$\Rightarrow$ TRUE, similarly with

$a = 1,\ b = 4,\ c = 7$

and $a = 2,\ b = 5,\ c = 8$

example

$\frac{a - b - 6}{9} = q$

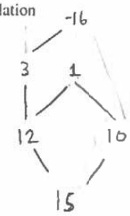
$\frac{|a - b| - 3}{9} = q$

c. **(1 points)** If we view " = " as a subset of $A \times A$, how many elements does "=" have? Do not list the elements, just tell me the cardinality of "=".

$$3^2 + 3^2 + 3^2 = 27$$

(ii) Let $A = \{-16, 1, 3, 10, 12, 15\}$ and $B = \{0, -3, -5, -9, -11, -12, -14, -19, -28, -31\}$. Let $a, b \in A$. Define "$\preceq$" on A so that $a \preceq b$ if $(b - a) \in B$. Then "$\preceq$" is a partial relation on A (DO NOT SHOW THAT)

a. **(4 points)** Draw the Hassee diagram of such relation



b. **(5 points)** By staring at the Hassee diagram, If possible, find

i. $12 \wedge 10$ 15
ii. $10 \wedge 3$ 15
iii. $1 \wedge 3$ 12
iv. $15 \vee -16$ -16
v. $1 \wedge -16$ 12
vi. $1 \vee 15$ 1
vii. $10 \vee 3$ 15
viii. ~~$10 \wedge -16$~~ 15
ix. Is there a $c \in A$ such that $a \preceq c$ for every $a \in A$? If yes, find c no such c
x. Is there an $m \in A$ such that $m \preceq a$ for every $a \in A$? If yes, find m $m = 15$

Tasneem Batool 71143.

Exam II: MTH 213, Spring 2018

52/52 Excellent

Ayman Badawi

(Bonus), (1 point). This is math 213 and my instructor name is Ayman Badawi. His office hours are on 4h oh. from to . We meet every, Sunday, Tuesday, and Thursday in Nab at am.

QUESTION 1. Imagine: then

(i) $O(\frac{x^{0.7}+3x^{0.2}-5}{x+7})$ equals to $= \frac{O(x^{0.7}+3x^{0.2}-5)}{O(x+7)} = \frac{x^{0.7}}{x} = x^{-0.3}$

4/4 (ii) $O(\sqrt[3]{x}+x)(x^2-x^{7/2}-2)$ equals to $= O(x^{1/3}+x).O(x^2-x^{3.5}-2)$

$= x \cdot x^{3.5} = x^{4.5}$

QUESTION 2. Imagine: The following Algorithm segment. Find the exact number of additions, multiplications, and subtractions that will be performed when the algorithm is executed. Then find the complexity of the Algorithm segment.

$m = 7; s = 9$

For $k := 3$ *to* (n^2+2)

$L = k * m + 2 * s - 6$ 4 op.

For $i := 1$ *to* $(k+1)$

$s = s + m^3 + i - k^2$ 6 op.

$s + m*m*m + i - k*k$

next i

next k

No. of outer loop iterations = $L = n^2+2-3+1 = n^2$.

at $L = 1.(k=3)$

outer loop op: 4

inner loop op $= (6)(k+1-1+1)$

$= (6)(4) = 24$.

total op $= 4 + 24 = 28$.

at $L = n^2$ $(k = n^2+2)$

outer loop op = 4

inner loop op $= (6)(k+1)$

$= (6)(n^2+2+1)$

$= (6)(n^2+3)$

$= 6n^2+18$.

Total op: $4 + 6n^2 + 18$

$= 6n^2 + 22$.

$\therefore$ Sum $= \frac{(28+6n^2+22)n^2}{2}$

$= \frac{(6n^2+50)n^2}{2}$

$= 3n^4 + 25n^2$.

Complexity:

$O(3n^4+25n^2) = n^4$.

8/8

QUESTION 3. 1) Imagine: there are 220 flowers in a shop and each flower is either red or green or yellow (nothing else). You heard someone saying: There are at least 10 flowers that have the same color. You smiled and you told yourself : Definitely this person never took Math 213 or this person in the shop must be Ayman Badawi, since his arithmetic is very week. So can you make a better statement that is always true and no one can improve it? i.e., your statement: There are at least so and so...that have the same color.

4/4 Ans: |Domain| = 220 $m = \lceil \frac{220}{3} \rceil = 74$.

|Co-domain| = 3

Result: There are at least 74 flowers of the same color.

2) Imagine: You travelled faster than the light and you discovered that only 727 persons were born between 2026-2031. Then there is a year where at most m persons were born in that year. Find the minimum value of m.

6 years

4/4 Ans: |Domain| = 727. $m = \lfloor \frac{727}{6} \rfloor = 121$.

|Codomain| = 6

4) Use your own ENGLISH LANGUAGE : Explain to me the meaning of the answer in part (2).

Ans: By treating the number of people born as the domain, and number of birth years as co-domain, the answer in part (2) suggests that in ~~a~~ every possible scenario of this function, there will be a year (betw. 2026 and 2031), where no more than 121 people were born. This number ~~best~~ is the best assumption/conclusion as opposed to any numbers ~~greater~~ than it. greater.

4/4

Excellent

Exam II: Discrete Math, MTH 213, Fall 2017

Ayman Badawi

$\frac{55}{55}$

M. Said (Perfect score, second time in a row)

QUESTION 1. Assume there are 550 persons in the main building of AUS. Then

(i) There are at least n persons who were born in the same month and on the same day of the week. What is the maximum value of n that we all are sure about?

2

D: {1, 2, 3, …, 550} $\xrightarrow{\text{onto}}$ R: {1, 2, …, 84}

it's an onto function where $|R| < |D|$ and we assume fare sharing $\Rightarrow$ by Pigeonhole Principle we have at least $n = \lceil \frac{550}{84} \rceil = \boxed{7}$ persons

we are sure also that were born on the same day on the same month

(ii) There must exist a day of the week such that no more than m persons were born on that day. what is the maximum value of m ?(hint: THINK! not difficult)

2

if all students were born on the same day of the week the we can have maximum $\boxed{m = 550}$ persons who were born on that day.

QUESTION 2. (i) 26 distinct numbers were chosen randomly. Then there are at least n numbers out of the chosen numbers that have the same unit digit. What is the maximum value of n that we all are sure about?

2

D: {1, …, 26} $\xrightarrow{\text{onto}}$ R: {0, 1, 2, 3, …, 9}

it's an onto function where $|R| < |D|$ and assuming fare sharing $\Rightarrow$ by Pigeonhole principle we have at least $n = \lceil \frac{26}{10} \rceil = \boxed{3}$ numbers that have the same unit digit

(ii) Assume 302 persons were in a party. Assume that the party started at 8pm and it ended at 2am. Then there are at least n persons in the party of the same sex. Find the maximum value of n that we all are sure about? (Hint: Smile!)

2

D: {1, 2, …, 302} $\xrightarrow{\text{onto}}$ R: {1, 2}

it's an onto function where $|R| < |D|$ and assuming fare sharing $\Rightarrow$ by Pigeonhole Principle we have at least $n = \lceil \frac{302}{2} \rceil = \boxed{151}$ Persons of the same sex

QUESTION 3. We have 7 holes labeled from 1 to 7 and we have 4 balls (red, blue, green, yellow). We need to put each ball in one hole. Find all possible ways?

2 $7C4 \times 4P4$

QUESTION 4. An electrical panel has six switches. How many ways can the switches be positioned up or down if four switches must be up and 2 switches must be down. (note order not important)

2 $6C4 \times 2C2$

QUESTION 5. How many 4-digit even numbers greater than 400 can be formed using the digits 1, 2, 3, 4, 5, and 6?

2

1 _ _ _
2 _ _ _
3 _ _ _
4 _ _ _
5 _ _ _
6 _ _ _

$X = \underline{x_1}\ \underline{x_2}\ \underline{x_3}\ \underline{x_4}$

$X = 6C1 \times 6C1 \times 6C1 \times 3C1 = 648$

QUESTION 6. If there are 3 buses and 4 cars, how many ways can the vehicles park in a line such that cars and buses alternate positions?(note order is important)

2

$3P3 \times 4P4$

QUESTION 7. (i) the Mickey-function $\frac{x^{0.5}+3x^{5/2}-5}{x+7}$ is $\Theta(X^{\frac{3}{2}})$ and it is $O(X)^{\frac{3}{2}}$

2

(ii) The Mickey-polynomial $\sqrt{x}(x^3 - x^{11/2} + 7)$ is $\Theta(X^6)$ and it is $O(X)^6$

2

$x^{\frac{7}{2}} - x^6 + 7$

QUESTION 8. Consider the following Algorithm segment. Find the exact number of additions, multiplications, and subtractions that will be performed when the algorithm is executed. Then find the order of the Algorithm segment.

$m = 7; s = 0$

For $k := 4$ *to* $n + 1$ → $n + 1 - 4 + 1 = n - 2$

For $i := 1$ *to* 10 → $10 - 1 + 1 = 10$

$s = s + m^3 + i - k^2$ 6

next i

$L = k + 2 * s - 6$ 3

next k

4.

for $k = 4$ . . . . $k = n + 1$

$(10 \times 6) + 3$ $(10 \times 6) + 3$

$\Rightarrow \frac{(n-2)(63+63)}{2} = \frac{126n - 252}{2}$

the number of multiplications, subtractions and addiors $= \boxed{63n - 126}$

the code is of order $\Theta(n)$ and $O(n)$